

AP Calculus BC

Implicit Differentiation

1) $xy^2 - 2x^3 = 2$ for $y \geq 0$

a) $x(2yy') + y^2 - 6x^2 = 0$
 $2xyy' + y^2 - 6x^2 = 0$
 $\frac{dy}{dx} = \frac{6x^2 - y^2}{2xy}$

b) $\frac{dy}{dx} \Big|_{(1,2)} = \frac{6-4}{4} = \frac{1}{2}$

$y-2 = \frac{1}{2}(x-1)$

c) $\frac{6x^2 - y^2}{2xy} = 0$
 $6x^2 - y^2 = 0$
 $6x^2 = y^2$

$x(6x^2) - 2x^3 = 2$
 $6x^3 - 2x^3 = 2$
 $4x^3 = 2$
 $x^3 = \frac{1}{2}$
 $x = \sqrt[3]{\frac{1}{2}}$

d) $\frac{d^2y}{dx^2} = \frac{(2xy)(12x - 2yy') - (6x^2 - y^2)(2xy' + 2y)}{(2xy)^2}$

$\frac{d^2y}{dx^2} \Big|_{(1,2)} = \frac{(4)(12 - 4(\frac{1}{2})) - (2)(5)}{(16)}$

2) $y^2 - x^2y = 6$

a) $2yy' - x^2y' - 2xy = 0$
 $2yy' - x^2y' = 2xy$
 $y' = \frac{2xy}{2y - x^2}$

b) $y' \Big|_{(1,3)} = \frac{6}{6-1} = \frac{6}{5}$
 $y-3 = \frac{6}{5}(x-1)$

c) $y^2 - 0 = 6$
 $y = \sqrt{6}$
 $\frac{dy}{dx} = \frac{2(0)\sqrt{6}}{2\sqrt{6} - 0} = 0$ $(0, \sqrt{6})$

d) $\frac{d^2y}{dx^2} = \frac{(2y - x^2)(2xy' + 2y) - (2xy)(2y' - 2x)}{(2y - x^2)^2}$
 $\frac{d^2y}{dx^2} \Big|_{(0, \sqrt{6})} = \frac{(2\sqrt{6})(2\sqrt{6}) - 0}{(2\sqrt{6})^2} = 1$